

ASYMPTOTIC UPPER BOUNDS ON THE SHADES OF t -INTERSECTING FAMILIES

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ABSTRACT. We examine the m -shades of t -intersecting families of k -subsets of $[n]$, and conjecture on the optimal upper bound on their cardinalities. This conjecture extends Frankl's General Conjecture that was proven true by Ahlswede–Khachatrian. From this we deduce the precise asymptotic upper bounds on the cardinalities of m -shades of $t(m)$ -intersecting families of $k(m)$ -subsets of $[2m]$, as $m \rightarrow \infty$. A generalization to cross- t -intersecting families is also considered.

1. INTRODUCTION

The paper [Hir08] was concerned with the dichotomy below of descriptive set theory. This dichotomy is aimed towards research on a fundamental question of set theoretic forcing, of whether Cohen and random forcing together form a basis for all nontrivial Souslin (i.e. “simply” definable) ccc (i.e. no uncountable antichains) posets (asked by Shelah in [She94]).

Dichotomy 1. *Every analytic (i.e. projection of a closed subset of the “plane”) family \mathcal{A} of infinitely branching subtrees of $\{0,1\}^{<\mathbb{N}}$ satisfies at least one of the following:*

- (a) *There exists a colouring $c : \{0,1\}^{<\mathbb{N}} \rightarrow \{0,1\}$ and a such that $S(n)$ is nonhomogeneous for c for all but finitely many $n \in \mathbb{N}$, for every $S \in \mathcal{A}$.*
- (b) *The poset (\mathcal{A}, \subseteq) has an uncountable antichain.*

It turned out that obtaining tight upper bounds on the m -shades of t -intersecting families of k -subsets of $[m]$, was relevant to dichotomy 1. The connection is described in lemma 1 below (cf. [Hir08] for details).

1.1. Shades. One of the basic notions in Sperner theory is the *shade* (also called *upper shadow*) of a set or a family of sets (see e.g. [And02],[Eng97]). For a subset x of a fixed set S , the shade of x is

$$(1) \quad \nabla(x) = \{y \subseteq S : x \subset y \text{ and } |y| = |x| + 1\},$$

and the shade of a family X of subsets of S is

$$(2) \quad \nabla(X) = \bigcup_{x \in X} \nabla(x).$$

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Recall that the m -shade (also called *upper m -shadow* or *shade at the m^{th} level*) of x is

$$(3) \quad \nabla_{\rightarrow m}(x) = \{y \subseteq S : x \subseteq y \text{ and } |y| = m\},$$

and $\nabla_{\rightarrow m}(X) = \bigcup_{x \in X} \nabla_{\rightarrow m}(x)$. We follow the Sperner theoretic conventions of writing $[m, n]$ for the set $\{m, m+1, \dots, n\}$ and $[n]$ for the set $[1, n] = \{1, \dots, n\}$.

We introduce the following notation for colouring sets with two colours. For a set S , let $\binom{S}{[m]}$ denote the collection of all colourings $c : S \rightarrow \{0, 1\}$ with $|c^{-1}(0)| = m$, i.e. $c^{-1}(0) = \{j \in S : c(j) = 0\}$. This is related to shades, because for all $c \in \binom{S}{[m]}$ and all $x \subseteq S$,

$$(4) \quad x \text{ is homogeneous for } c \text{ iff } c^{-1}(0) \in \nabla_{\rightarrow m}(x) \text{ or } c^{-1}(1) \in \nabla_{\rightarrow |S|-m}(x).$$

When a nonhomogeneous colouring is desired, it is most efficient to use colorings in $\binom{S}{[m]}$ for $|S| = 2m$. Equation (4) immediately gives us:

Lemma 1. *Suppose X is a family of subsets of $[2m]$. Then*

$$\left| \left\{ c \in \binom{[2m]}{[m]} : \exists x \in X \text{ } x \text{ is homogeneous for } c \right\} \right| \leq 2|\nabla_{\rightarrow m}(X)|$$

(the shades are with respect to $S = [2m]$).

2. UPPER BOUNDS

Recall that a family \mathcal{A} of sets is t -intersecting if $|E \cap F| \geq t$ for all $E, F \in \mathcal{A}$; and a pair $(\mathcal{A}, \mathcal{B})$ of families of subsets of some fixed set are *cross- t -intersecting* if

$$(5) \quad |E \cap F| \geq t \quad \text{for all } E \in \mathcal{A}, F \in \mathcal{B}.$$

Thus \mathcal{A} is t -intersecting iff $(\mathcal{A}, \mathcal{A})$ is cross- t -intersecting.

We use the standard notation $\binom{S}{k}$ to denote the collection of all k -subsets of S , and hence $\binom{[n]}{k}$ denotes the collection of all subsets of $[n]$ of cardinality k . Let $I(n, k, t)$ denote the family of all t -intersecting subfamilies of $\binom{[n]}{k}$ (where $t \leq k \leq n$). Define the function

$$(6) \quad M(n, k, t) = \max_{\mathcal{A} \in I(n, k, t)} |\mathcal{A}|.$$

The investigation into the function M and the structure of the maximal families was initiated by Erdős–Ko–Rado in 1938, but not published until [EKR61]. In this paper, they gave a complete solution for the case $t = 1$, and posed what became one of the most famous open problems in this area. The following so called *4m-conjecture* for the case $t = 2$:

$$(7) \quad M(4m, 2m, 2) = \frac{1}{2} \left(\binom{4m}{2m} - \binom{2m}{m}^2 \right).$$

We briefly explain the significance of the right hand side expression. Define families

$$(8) \quad \mathcal{F}_i(n, k, t) = \left\{ F \in \binom{[n]}{k} : |F \cap [t+2i]| \geq t+i \right\} \quad \text{for } 0 \leq i \leq \frac{n-t}{2}.$$

Clearly each $\mathcal{F}_i(n, k, t)$ is t -intersecting. In the special case where $n = 2k = 2m$ and $t = 2s$, we can easily compute the cardinality of the corresponding \mathcal{F}_i using

the fact that $[2m] \setminus F$ is an m -set for all $F \in \binom{[2m]}{m}$, i.e. $|\mathcal{F}_i(2m, m, 2s)|$ equals

$$(9) \quad \frac{1}{2} \left(\binom{2m}{m} - \sum_{j=-(s-1)}^{s-1} \binom{2(s+i)}{s+i+j} \binom{2m-2(s+i)}{m-(s+i+j)} \right).$$

Then plugging in $m := 2m$ (i.e. $2m$ for m), $s := 1$ and $i := m-1$ we see that the right hand side of equation (7) is equal to the cardinality of $\mathcal{F}_{m-1}(4m, 2m, 2)$. The $4m$ -conjecture was generalized by Frankl in 1978 ([Fra78]) as follows: For all $1 \leq t \leq k \leq n$,

$$(10) \quad M(n, k, t) = \max_{0 \leq i \leq \frac{n-t}{2}} |\mathcal{F}_i(n, k, t)|.$$

In 1995, the general conjecture was proven true by Ahlswede–Khachatrian in [AK97], where they moreover established that the optimal families in $I(n, k, t)$ are equal to one of the families $\mathcal{F}_i(n, k, t)$ up to a permutation of $[n]$. This finally settled the $4m$ -conjecture, and moreover proved that the maximal family in $I(4m, 2m, 2)$ is isomorphic to $\mathcal{F}_{m-1}(4m, 2m, 2)$.

For reasons alluded to in lemma 1, it is upper bounds on the cardinality of the shades of t -intersecting families that we are interested in, rather than upper bounds on the families themselves. While there are numerous results giving lower bounds on the size of shadows/shades, upper bounds seem to be rather scarce. Perhaps this is because they are not very good. For example, the following is from [Kos89], where 2^S denotes the power set of S .

Theorem 1 (Kostochka, 1989). *Suppose that $\mathcal{A} \subseteq 2^{[n]}$ is a Sperner family. Then $\nabla(\mathcal{A}) \leq 0.724 \cdot 2^n$.*

Moreover, the best upper bound is known to be greater than $0.5 \cdot 2^n$.

However, in the case of t -intersecting families, the shade is also t -intersecting. Define for $1 \leq t \leq k \leq m \leq n$,

$$(11) \quad M_0(n, m, k, t) = \max_{\mathcal{A} \in I(n, k, t)} |\nabla_{\rightarrow m}(\mathcal{A})|,$$

i.e. $M_0(n, m, k, t)$ is the maximum size of the m -shade of a t -intersecting family of k -subsets of $[n]$. Thus we have

$$(12) \quad M_0(n, m, k, t) \leq M(n, m, t),$$

but this is not optimal. Indeed we make the following easy observations.

Lemma 2. *For all $1 \leq t \leq k \leq m \leq n$,*

- (a) $\mathcal{F}_i(n, k, t) = \emptyset$ for all $i > k - t$,
- (b) $\nabla_{\rightarrow m}(\mathcal{F}_i(n, k, t)) = \mathcal{F}_i(n, m, t)$ for all $0 \leq i \leq \min(k - t, \frac{n-t}{2})$.

This leads to the following conjecture.

Conjecture 1. $M_0(n, m, k, t) = \max_{0 \leq i \leq \min(k-t, \frac{n-t}{2})} |\mathcal{F}_i(n, m, t)|$.

Note that conjecture 1 is correct so long as the optimal families that we are taking the m -shades of are among the $\mathcal{F}_i(n, k, t)$.

2.1. Asymptotic behaviour. Not surprisingly, for the purpose of our set theoretic dichotomy we were interested in the asymptotic behaviour of the upper bounds, i.e. as $n \rightarrow \infty$. Furthermore, we were interested in the m -shade of subsets of $[2m]$, i.e. $n = 2m$. We would like something to the effect that the maximum proportion of the m -shade to the entire family $\binom{[2m]}{m}$ goes to 0; symbolically,

$$(13) \quad \lim_{\substack{m \rightarrow \infty \\ t(m) \leq k(m) \leq m}} \frac{M_0(2m, m, k(m), t(m))}{\binom{2m}{m}} = 0.$$

However, this is false, because for example the optimal family for the $4m$ -conjecture (cf. equation (7)) gives us $\lim_{m \rightarrow \infty} M_0(2m, m, m, 2) / \binom{2m}{m} = \frac{1}{2}$. In fact, equation (13) fails whenever $t(m)$ is bounded:

$$(14) \quad \lim_{m \rightarrow \infty} \frac{M_0(2m, m, t, t)}{\binom{2m}{m}} \geq \frac{1}{2^t}.$$

This can be seen by noting that $\nabla_{\rightarrow m}(\mathcal{F}_0(2m, t, t)) = \mathcal{F}_0(2m, m, t)$, and that $|\mathcal{F}_0(2m, m, t)| = \binom{2m-t}{m-t}$.

Moreover, as we shall demonstrate, even if $t(m) \xrightarrow{m \rightarrow \infty} \infty$, equation (13) can still fail if $t(m)$ is too small compared with $k(m)$. Henceforth, we shall make the simplification that $k(m) = o(m)$, i.e. $\lim_{m \rightarrow \infty} k(m) / m = 0$; this was the only case used in our application.

Before proceeding further, recall the de Moivre–Laplace theorem (cf. [Usp37]), roughly stating that the binomial series of $(p+q)^n$ has most of the sum concentrated in the order of \sqrt{n} terms around the center: For all $0 \leq a, b < \infty$,

$$(15) \quad \lim_{n \rightarrow \infty} \sum_{j=-\lfloor a\sqrt{n/2} \rfloor}^{\lfloor b\sqrt{n/2} \rfloor} \frac{\binom{2n}{n+j}}{4^n} = \Phi(b) - \Phi(-a),$$

where

$$(16) \quad \Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t e^{-x^2/2} dx$$

is the cumulative distribution function of the standard normal distribution. Recall that

$$(17) \quad \Phi(t) - \Phi(-t) = 2\Phi(t) - 1.$$

We write $f \sim g$ to indicate asymptotic equality, i.e. $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 1$. The theorem also tells us that

$$(18) \quad \frac{\binom{2n}{n+j}}{4^n} \sim \frac{e^{-(j/\sqrt{n/2})^2/2}}{\sqrt{\pi n}},$$

and moreover that the convergence is uniform over $j / \sqrt{n/2}$ in the range $[a, b]$.

We derive a version of (15) for the identity $\binom{2n}{n} = \sum_{j=-k}^k \binom{2k}{k+j} \binom{2(n-k)}{(n-k)-j}$.

Lemma 3. *Assume $k(n) = o(n)$ and $\lim_{n \rightarrow \infty} k(n) = \infty$. Then*

$$(19) \quad \lim_{n \rightarrow \infty} \sum_{j=-\lfloor a\sqrt{k(n)/2} \rfloor}^{\lfloor b\sqrt{k(n)/2} \rfloor} \frac{\binom{2k(n)}{k+j} \binom{2(n-k(n))}{(n-k(n))-j}}{\binom{2n}{n}} = \Phi(b) - \Phi(a).$$

Proof. By equation (18) and the assumptions on k , we have $\binom{2k(n)}{k(n)+j} \sim \frac{4^{k(n)}}{\sqrt{\pi k(n)}} \cdot e^{-(j/\sqrt{k(n)/2})^2/2}$ and $\binom{2(n-k(n))}{(n-k(n))-j} \sim \frac{4^{n-k(n)}}{\sqrt{\pi(n-k(n))}} \cdot e^{-(j/\sqrt{(n-k(n))/2})^2/2}$, with uniform convergence for $j / \sqrt{k(n)/2} \in [a, b]$. Therefore,

$$(20) \quad \binom{2k(n)}{k(n)+j} \binom{2(n-k(n))}{(n-k(n))-j} \sim \frac{4^n e^{-(j/\sqrt{k(n)/2})^2/2}}{\pi \sqrt{k(n)(n-k(n))}}$$

with uniform convergence. Changing variables then gives

$$(21) \quad \sum_{j=-\lfloor a\sqrt{k(n)/2} \rfloor}^{\lfloor b\sqrt{k(n)/2} \rfloor} \binom{2k(n)}{k(n)+j} \binom{2(n-k(n))}{(n-k(n))-j} \sim \frac{4^n}{\pi \sqrt{2n}} \int_a^b e^{-x^2/2} dx,$$

and the right hand side expression is equal to $\frac{4^n}{\sqrt{\pi n}}(\Phi(b) - \Phi(a))$ which is asymptotically equal to $\binom{2n}{n}(\Phi(b) - \Phi(a))$ by equation (18) with $j = 0$, as required. \square

Lemma 4. Let $c > 0$. Supposing $k(m) = o(m)$ and $\lim_{m \rightarrow \infty} k(m) = \infty$,

$$(22) \quad \lim_{m \rightarrow \infty} \frac{|\mathcal{F}_{k(m)}(2m, m, c\sqrt{k(m)})|}{\binom{2m}{m}} = \frac{1}{\sqrt{2\pi}} \int_{c/\sqrt{2}}^{\infty} e^{-x^2/2} dx.$$

Proof. Setting $s(m) := c\sqrt{k(m)} / 2$ and $i(m) := k(m)$, since $\sqrt{s(m) + i(m)} = \sqrt{k(m)} + o(\sqrt{k(m)})$, by lemma 3 and (17)

$$(23) \quad \lim_{m \rightarrow \infty} \frac{\sum_{j=-(s(m)-1)}^{s(m)-1} \binom{2(s(m)+i(m))}{s(m)+i(m)+j} \binom{2m-2(s(m)+i(m))}{m-(s(m)+i(m)+j)}}{\binom{2m}{m}} = 2\Phi(c/\sqrt{2}) - 1.$$

Hence by equation (9), the limit in (22) is equal to $\frac{1}{2}(1 - (2\Phi(c/\sqrt{2}) - 1)) = 1 - \Phi(c/\sqrt{2})$ as required. \square

For an infinite $A \subseteq \mathbb{N}$ we let $e_A : \mathbb{N} \rightarrow \mathbb{N}$ denote the strictly increasing enumeration of A .

Corollary 5. Assume $k(m) = o(m)$. Suppose there exists $c > 0$ such that

$$(24) \quad t(m) \leq c\sqrt{k(m)} \quad \text{for infinitely many } m.$$

Then $\limsup_{m \rightarrow \infty} M_0(2m, m, k(m), t(m)) / \binom{2m}{m} > 0$.

Proof. Let $A = \{m \in \mathbb{N} : t(m) \leq c\sqrt{k(m)}\}$, which is infinite by assumption. If $\lim_{m \rightarrow \infty} k \circ e_A(m) \neq \infty$, then $t(m)$ is bounded on an infinite subset $B \subseteq A$, and the argument in equation (14) shows that $M_0(2m, m, k(m), t(m)) / \binom{2m}{m}$ is bounded away from 0 on B . Otherwise, we can choose k' so that $k'(m) = k(m)$ for all $m \in A$, $\lim_{m \rightarrow \infty} k'(m) = \infty$ and $k'(m) = o(m)$.

Put $k''(m) = \max(0, k'(m) - c\sqrt{k'(m)})$. Obviously $\lim_{m \rightarrow \infty} k''(m) = \infty$ and $k''(m) = o(m)$. And for all $m \in A$ with $k''(m) > 0$, $\mathcal{F}_{k''(m)}(2m, k(m), c\sqrt{k'(m)})$ is a $t(m)$ -intersecting family of $k(m)$ -subsets of $[2m]$, and furthermore its m -shade is $\mathcal{F}_{k''(m)}(2m, m, c\sqrt{k'(m)})$ by lemma 2. The result thus follows from lemma 4 with $k := k''$. \square

Avoiding the example of corollary 5, we arrive at the following optimal conjecture, i.e. there is no room for improvement on $t(m)$ by corollary 5.

Conjecture 2. $\lim_{m \rightarrow \infty} \frac{M_0(2m, m, k(m), t(m))}{\binom{2m}{m}} = 0,$

whenever $k(m) = o(m)$, $\lim_{m \rightarrow \infty} k(m) = \infty$ and $\lim_{m \rightarrow \infty} \frac{t(m)}{\sqrt{k(m)}} = \infty$.

Let us show that conjecture 2 is a consequence of conjecture 1.

Lemma 6. Assume $t(m), l(m) = o(m)$ and $\lim_{m \rightarrow \infty} t(m) + l(m) = \infty$. Supposing that $\lim_{m \rightarrow \infty} \frac{t(m)}{\sqrt{l(m)}} = \infty$,

$$(25) \quad \lim_{m \rightarrow \infty} \frac{|\mathcal{F}_{l(m)}(2m, m, t(m))|}{\binom{2m}{m}} = 0.$$

Proof. Setting $s(m) := t(m) / 2$ and $i(m) := l(m)$, by our assumptions on t and l , lemma 3 applies yielding

$$(26) \quad \lim_{m \rightarrow \infty} \frac{\sum_{j=-(s(m)-1)}^{s(m)-1} \binom{2(s(m)+i(m))}{s(m)+i(m)+j} \binom{2m-2(s(m)+i(m))}{m-(s(m)+i(m)+j)}}{\binom{2m}{m}} = 1.$$

The result now follows from equation (9). \square

Corollary 7. If conjecture 1 is correct then so is conjecture 2.

Proof. Assume $k(m) = o(m)$, $k(m) \rightarrow \infty$ and $\lim_{m \rightarrow \infty} t(m) / \sqrt{k(m)} = \infty$; note that we are implicitly assuming that $t(m) \leq k(m)$ for all m (so that $M_0(2m, m, k(m), t(m))$ makes sense). Define k' so that for all m , $0 \leq k'(m) \leq k(m) - t(m)$ and

$$(27) \quad \max_{0 \leq i \leq k(m)-t(m)} |\mathcal{F}_i(2m, m, t(m))| = |\mathcal{F}_{k'(m)}(2m, m, t(m))|.$$

Setting $l(m) := k'(m)$, it is clear that t and l satisfy the hypotheses of lemma 6. Now

$$(28) \quad \lim_{m \rightarrow \infty} \frac{M_0(2m, m, k(m), t(m))}{\binom{2m}{m}} = \lim_{m \rightarrow \infty} \frac{|\mathcal{F}_{l(m)}(2m, m, t(m))|}{\binom{2m}{m}} = 0,$$

where the first equality is by conjecture 1, and the second by lemma 6. \square

3. CROSS- t -INTERSECTING FAMILIES

It turns out that for our application we needed upper bounds on the size of shades of cross- t -intersecting families (cf. equation (5)). Let $C(n, k, l, t)$ be the collection of all pairs $(\mathcal{A}, \mathcal{B})$ of cross- t -intersecting families, where $\mathcal{A} \subseteq \binom{[n]}{k}$ and $\mathcal{B} \subseteq \binom{[n]}{l}$. Then the cross- t -intersecting function corresponding to M is defined by

$$(29) \quad N(n, k, l, t) = \max_{(\mathcal{A}, \mathcal{B}) \in C(n, k, l, t)} |\mathcal{A}| \cdot |\mathcal{B}|.$$

There are a number of results on cross- t -intersecting families in the literature; however, the state of knowledge seems very meager compared with t -intersecting families. The following theorem, proved in [MT89], is the strongest result of its kind that we were able to find.

Theorem 2 (Matsumoto–Tokushige, 1989). $N(n, k, l, 1) = \binom{n-1}{k-1} \binom{n-1}{l-1}$ whenever $2k, 2l \leq n$.

Note that this corresponds to case $t = 1$ of the Erdős–Ko–Rado Theorem, proved back in 1938. It is also conjectured that the EKR Theorem does generalize:

Conjecture 3. $N(n, k, l, t) = \binom{n-t}{k-t} \binom{n-t}{l-t}$ for all $n \geq n_0(k, l, t)$.

Generalizing the families \mathcal{F}_i , we define

$$(30) \quad \mathcal{G}_{ij}(n, k, t) = \left\{ F \in \binom{[n]}{k} : |F \cap [t + i + j]| \geq t + i \right\}$$

for $0 \leq i + j \leq n - t$; e.g. $\mathcal{F}_i(n, k, t) = \mathcal{G}_{ii}(n, k, t)$. Observe that:

Proposition 8. $(\mathcal{G}_{ij}(n, k, t), \mathcal{G}_{ji}(n, l, t))$ is cross- t -intersecting whenever $0 \leq i + j \leq n - t$.

We make the following conjecture, generalizing the Ahlswede–Khachatrian Theorem (i.e. that Frankl’s General Conjecture is true, cf. equation (10)).

Conjecture 4. $N(n, k, l, t) = \max_{0 \leq i+j \leq n-t} |\mathcal{G}_{ij}(n, k, t)| \cdot |\mathcal{G}_{ji}(n, l, t)|$.

Moreover, up to a permutation of $[n]$, the optimal cross- t -intersecting family is of the form $(\mathcal{G}_{ij}(n, k, t), \mathcal{G}_{ji}(n, l, t))$ for some i, j .

Generalizing M_0 , we define the maximum size $N_0(n, m_k, m_l, k, l, t)$ of the product of the m_k -shade with the m_l -shade of a pair of cross- t -intersecting families of k -subsets and l -subsets of $[n]$, respectively:

$$(31) \quad N_0(n, m_k, m_l, k, l, t) = \max_{(\mathcal{A}, \mathcal{B}) \in \mathcal{C}(n, k, l, t)} |\nabla_{\rightarrow m_k}(\mathcal{A})| \cdot |\nabla_{\rightarrow m_l}(\mathcal{B})|.$$

For purposes of our dichotomy, we were exclusively interested in the numbers $N_0(2m, m, m, k, k, t)$. Thus we define

$$(32) \quad N_1(n, m, k, t) = N_0(n, m, m, k, k, t).$$

Corresponding to lemma 2 we have:

Lemma 9. For all $1 \leq t \leq k \leq m \leq n$,

- (a) $\mathcal{G}_{ij}(n, k, t) = 0$ for all $i > k - t$,
- (b) $\nabla_{\rightarrow m}(\mathcal{G}_{ij}(n, k, t)) = \mathcal{G}_{ij}(n, m, t)$ for all $0 \leq i \leq k - t$ with $i + j \leq n - t$.

Then corresponding to conjecture 1 we have:

Conjecture 5. $N_0(n, m_k, m_l, k, l, t) = \max_{\substack{0 \leq i \leq k-t \\ 0 \leq j \leq l-t \\ i+j \leq n-t}} |\mathcal{G}_{ij}(n, m_k, t)| \cdot |\mathcal{G}_{ji}(n, m_l, t)|$.

Finally, we arrive at the corresponding asymptotic conjecture.

Conjecture 6. Assume $k(m) = o(m)$ and $\lim_{m \rightarrow \infty} k(m) = \infty$. Suppose that $\lim_{m \rightarrow \infty} \frac{t(m)}{\sqrt{k(m)}} = \infty$. Then

$$(33) \quad \lim_{m \rightarrow \infty} \frac{\sqrt{N_1(2m, m, k(m), t(m))}}{\binom{2m}{m}} = 0.$$

We expect that the argument for corollary 7 will generalize, so that one can obtain conjecture 6 and a consequence of conjecture 5.

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